Euler's Mechanics as Opposition to Leibnizian Dynamics[1]

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1. Introduction

Leonhard Euler, the notable mathematician in the eighteenth century, is also famous for his contributions to mechanics. It would not be an exaggeration to say that what we call "Newtonian mechanics" was established by Euler, rather than Newton. However, it appears to me that historians of mechanics have failed to identify what was truly innovative in Euler's thought. This is mainly because, I suppose, they didn't pay enough attention to a Leibnizian context. My argument is not that Euler was a direct descendant of Leibniz. Rather, I try to show that Euler's conceptions of mechanics were in opposition to the Leibnizian doctrine.

My presentation consists of three parts. First, I'll give a very short introduction to the mechanics in the eighteenth century and explain why the Leibnizian tradition matters. Next, I'll give a brief sketch of Leibnizian dynamics and its reception in the early period of that century. In the section four, then, I turn to some of Euler's earliest writings on mechanics.

2. Eighteenth-Century Mechanics and Leibnizian Tradition

Under the term of "eighteenth-century mechanics," I mean, roughly speaking, the science of motion and force between Newton's *Principia* (1687) and Lagrange's *Mechanique Analytique* (1788). In the course of this one-hundred years, the mathematical language of mechanics changed from geometry into algebraic analysis, and at the same time several "general principles" were proposed, including the conservation of *vis viva* (kinetic energy), the principle of least action, "d'Alembert's principle," and what we now call the equations of motion.

It was mainly mathematicians on the European Continent who solved the mechanical problems proposed in *Principia* and other kinds of motion with Leibnizian calculus. Among those mathematicians we find Varignon, the Bernoullis (brothers of Jacob and Johann, and Daniel), Euler, d'Alembert, and Lagrange. For example, Jacob Hermann and Johann Bernoulli are famous for their contributions to what is called the inverse problem of central force, that is, to determine a motion of bodies which are acted by attractive forces.

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[1] Paper read at History of Science Society 2014 Annual Meeting, Chicago, Nov. 9, 2014.
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So, a large part of the eighteenth-century mechanics was Leibnizian in terms of mathematics. But how about its physical aspect? Did they just borrow the Leibnizian calculus to enhance the alleged "Newtonian" mechanics? Or, did they consider motion and force in another, "Leibnizian" manner? I argue that there was really a kind of "Leibnizian" dynamics.

In fact, some scholars have paid attention to the Leibnizian tradition of dynamics in their studies on Kant. They argues that Kant's understanding of mechanics was not Newtonian, but Leibnizian. I find this intellectual background to Kant also important for Euler. Now I'll turn to the Leibnizian doctrine and its influence.

3. Leibnizian Dynamics

The science of dynamics, and the term of "dynamics" itself, was an invention of Leibniz. It was actually proposed in his effort to establish a new philosophy of nature. Here I stress just one thing: Leibniz identified "force" as essential for bodies. Let us see his basic ideas stated in his "Specimen dynamicum" (1695), the most important text on the subject.

In this article, Leibniz classifies "force" in two ways. One classification is based on the distinction between "primitive" force, which is in the domain of metaphysics, and "derivative" force, which is related to physics. Another distinction is about the opposition of "active" and "passive" force. Thus one has four kinds of "force" in total.

Leibnizian "dynamics" involves the "derivative" and "active" force, and Leibniz classifies this kind of force further into two species: *vis viva* (living force) and *vis mortua* (dead force). *Vis viva* is associated with bodies in actual motion, while *vis mortua* is related to the situation that motion doesn't exist yet, but that only "solicitation" for motion exists. Furthermore, Leibniz argued that *vis viva* of a body was proportional to its mass and squared velocity, while *vis mortua* was in direct proportion to velocity. One might see, in these discussions, an origin of modern notion of kinetic and potential energy.

It's interesting to note that Leibniz considered his "dynamics" as a sort of opposition to "mechanics" in its traditional sense, that is, science of machines. The subject of this science is, basically, the equilibrium of bodies or mechanical powers. Thus, according to Leibniz, "mechanics" only involves *vis mortua*, and *vis viva* is out of the scope. This is why Leibniz declared his "dynamics" as a "new" science, which considers the force of bodies in motion. As you may find, this understanding of "dynamics" is different from modern one.

The dynamics of Leibniz, or his ideas of *vis viva* and *vis mortua*, were accepted by some mathematicians and philosophers in the early eighteenth-century. Especially notable among them were Christian Wolff, Jacob Hermann and Johann Bernoulli.

Christian Wolff was the most influential philosopher in the German speaking world between Leibniz and Kant. With regard to dynamics, he contributed an article titled "Principia dynamica" to the Academy of St. Petersburg, which was published in 1728. Apart from the writings by Leibniz himself, the contribution seems to be the first publication that bears "dynamics" in its title. Wolff also explained concepts of *vis viva* and *vis mortua* in his *Elements of Universal Mathematics* (1713) and his *Mathematical Lexicon* (1716). These works contained the earliest descriptions about Leibnizian concepts of force.

The second person from the Leibnizian tradition is Jacob Hermann. He was a celebrated mathematician and a supporter of Leibnizian calculus. His reception of *vis viva* and *vis mortua* can be seen in *Phoronomia* (1716), his main work on force and motion. In its introductory section, Hermann explains Leibnizian understandings of force as well as some fundamental concepts such as mass and time. Furthermore, he composed an article on the measure of force (pub. 1728) and supported for the Leibnizian theory.

Finally, Johann Bernoulli entered the scene as a powerful proponent of Leibnizian doctrine. His *Discours*, published in Paris in 1727, was one of the most influential texts of mechanics in this century. Like Wolff and Hermann, Bernoulli gave an account of *vis viva* and *vis mortua* and argued for the Leibnizian measure of them. Bernoulli's *Discours* was the first detailed account of the Leibnizian concepts written in French, and it brought a heated debate at the Academy of Sciences of Paris.

At this point, I want to emphasize once again that Leibnizian dynamics was about force of bodies, and that all those figures I mentioned assumed two kinds of force. Taking only one example, Wolff explains as follows:

> "Vis Motrix" [Motive Force] or simply "Vis" [Force] is what produces motion. It is said "viva" [living], if it produces actual motion or connected with actual motion, such as that in a falling globe. Instead, it is called "mortua" [dead], if it tries to produce motion, indeed, but in fact not produce actual motion yet, such as that in a globe suspended by strings.

Similar accounts can be found in Hermann's and Bernoulli's writings. These conceptions of force makes their "dynamics" different from what is called Newtonian mechanics. The point is that even the representative mathematicians on the Continent didn't think in the modern framework of mechanics.

Then, it's interesting to see that those supporters for Leibnizian dynamics were all related to Euler. Bernoulli was the mentor of Euler at Basel, and when Euler moved from Basel to Petersburg, Euler visited Wolff. Hermann was an older colleague at the academy of Petersburg. Then, it seems natural to make this question: was Euler Leibnizian, too?

4. Euler's Attitude to Leibnizian Concepts

In 1727, Euler (then twenty-years old) applied for a chair of physics at the University of Basel. Although it was unsuccessful, he composed a dissertation on sound for this application, and to this work Euler added some "attachments." One of them asserts that the force of moving bodies is proportional to mass and squared velocity. Euler gives no explanation about this, but the statement clearly shows he did support for Leibnizian measure of *vis viva*.

We can see Euler's involvement with Leibnizian concepts more clearly in his early manuscripts. These materials were already published in the nineteenth and twentieth centuries, but I didn't find any detailed study on these items. I'll discuss some of them.

The first material is "True Reasoning for Estimating Forces." This was apparently composed when Euler was a student at Basel. In this article, Euler tries to prove the validity of Leibnizian measure of force. Euler's argument takes a form of syllogism: force of bodies should be estimated by its effect; effects of moving bodies is proportional to mass and squared velocity; therefore force of bodies is proportional to mass and squared velocity. I don't enter into their details, but please note that Euler explicitly talks about "force of bodies" and its measure.

In another manuscript (c. 1728-1731), Euler discusses *vis viva* and *vis mortua*. According to Euler, there are two modes in which bodies receive motion. The one is by *potentia* (power), that is "press or pull," which Euler associates with *vis mortua*. The examples include bodies connected with springs, iron near to magnets, and bodies with weight. The other mode is by the impact of moving bodies, and in this case the motion is obtained through *vis viva*. We may regard Euler's *potentia* as what we now call force, but he still admits *vis viva* as force of moving bodies. Furthermore, in this manuscript Euler discusses the measures of these two kinds of force as well.

As seen from these materials, Euler accepted Leibnizian conceptions of force at first. This is natural because he originally grew up in a Leibnizian culture. At the same time, however, Euler introduced another concept, *potentia*. And it was with this concept that Euler began to establish his theoretical system of mechanics.

One of the most remarkable examples is his article on collision of bodies. In 1731 Euler presented his first analysis of this subject. We have just seen that Euler associated collision of moving bodies with *vis viva*. Nevertheless, in this article, one can find neither "vis viva" nor "force of bodies." Instead, Euler based his whole analysis on *potentia*, which in turn was represented by imaginary springs. In modern physical terms, Euler describes the interaction between the two colliding bodies as a pressure by the spring, and he sets equations of motion for these bodies. By using this conceptual device, Euler was able to discuss the collision of bodies only through *potentia*, without referring to *vis viva*.

Still another manuscript will further reveal the point. The present material is assumed to be a lecture note at the Petersburg Academy, and given a title of "Lectures on Statics" by the editor of manuscripts. But Euler declares, at the beginning of the lecture, that he is going to explain the science of motion. What happens? In fact, Euler says as follows:

> Before Mechanics, however, another discipline, that is "Statics," is required. It treats of *potentia* [power], its comparison and its equilibrium. For without that one can make little progress in explaining motion of bodies, because the origin and production of motion should be derived from the nature of *potentia*. For us, therefore, the two sciences are to be studied: Statics and Mechanics.

This conception of mechanics was materialized in Euler's two-volume *Mechanica* (1736). As its subtitle (*Science of Motion Analytically Exposed*) shows, under "mechanics" Euler means the science of motion. And his treatment was based on the concept from statics. In his *Mechanica*, Euler employs *potentia*, rather than *vis*. Usually these two terms are both translated into "force" in English, but I suppose that this difference was essential for Euler.

Later, in 1744, Euler made clearer his critical attitude to Leibnizian doctrine. According to Euler, "the Leibnizians" maintains that *vis viva* (impact) and *vis mortua* (press) are totally different. But, in reality, "force of percussion is nothing but the operation of variable pressure …," he says. We have seen the origins of this criticism already appeared in Euler's earlier writings.

5. Conclusions

Now I'll summarize my points. First, I explained "dynamics" was originally a science of force of bodies, the science of *vis viva* in particular. According to Leibniz, this science was new because the ancient science of machines or "mechanics" was a science of *vis mortua*. Such understandings of force and science were promoted by some Leibnizian scholars, including Wolff, Hermann and Bernoulli. This was also the intellectual background to Euler, and he supported for the doctrine at first. But at an early stage of his career, Euler developed his own version of mechanics. Its characteristic feature was the concept of *potentia*, mechanical force treated in statics. Euler's mechanics was a science of motion based on statics, thus uniting two sciences.

Incidentally, Johann Bernoulli, the former mentor of Euler, appeared not so happy with this innovation. After obtaining Euler's *Mechanica*, he wrote to Euler and suggested that "dynamics" would have been better for the title of the book. According to Bernoulli, "mechanics" means the science of *vis mortua*, and for the science of *vis viva* we have a proper name of "dynamics." This was the Leibnizian understanding, and Euler's new mechanics rejected such a dualism of force and science. Thus we will have a better understanding of Euler's mechanics, if it is contextualized in the Leibnizian dynamics.