Euler's Principle of Least `Effort': Development and Interpretation

ARIGA Nobumichi (National Museum of Nature and Science, Japan)*

Paper Presentation at the 24th International Congress of History of Science, Technology and Medicine
University of Manchester (UK), Jul. 25, 2013

SUMMARY

Leonhard Euler's contribution to variational mechanics is commonly associated with his mathematical formulation of the principle of least action, which states that the "action" (integral of the mass multiplied by velocity along the path) is a minimum in physical processes. Although the second appendix to his *Methodus Inveniendi* (1744), where the formulation of that principle was first given, is traditionally regarded as Euler's main work in this field, historical studies have suggested that his original ideas about an optimal principle of nature were somewhat different. In this presentation I attempt to show how Euler arrived at and interpreted his own principle, which might better be called the principle of least "effort."

Euler began his study of variational principles in the early 1740s, inspired by Daniel Bernoulli. Euler found two such principles, one for elastic curves and the other for projectile motion, and published them in two appendices to his *Methodus Inveniendi*. Although Euler was convinced that nature was governed by some principle of optimization, he had no idea how to find the quantity which should be the minimum for each case. In the latter half of the 1740s, however, he recognized that any minimum quantity could be regarded as a special form of a general quantity he called "effort" (nearly corresponding to potential energy). He got this idea mainly from Maupertuis's 1740 work on statics, and confused it with Maupertuis's later works on the principle of least action. From Euler's perspective, the minimum principle in mechanics was not of "action" but of "effort," and it was more about statics than dynamics.

Statics was, as Euler conceived, the science of mechanical forces. Euler interpreted his principle of least "effort" as a consequence of the nature of those forces. The basis of this interpretation was his conception of force represented by imaginary springs, which endeavor to extend as much as possible. Euler gave such a descripion in his articles on the least-action principle, and it appears also in his posthumous work on statics. Moreover, Euler almost from the beginning of his career had employed such a spring-model to discuss the collision of bodies (this idea was probably inherited from Johann Bernoulli). Euler's understanding of the principle of least "effort" was, therefore, linked to his "metaphysical" conception of force, which explained why minimum properties were observed in nature.

* MAIL: ariga.nobumichi@gmail.com